A LINEAR PROGRAMMING MODEL FOR NAVAL OFFICER DISTRIBUTION

by

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THESIS

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ABSTRACT

The Officer Distribution Plan (ODP) has been developed by the Navy in an attempt to provide for the most equitable distribution of officer resources throughout the naval establishment. This thesis demonstrates the feasibility of constructing a linear programming model to represent the U. S. Navy officer personnel system for the purpose of developing the ODP and which can do in minutes what now takes months. Constraints in the model represent officer category limitations, activity configuration and billet eligility. A measure of billet fill effectiveness is developed through the assignment of penalty cost for perfect fills, imperfect fills, unassigned resources and unfilled and overfilled billets based on inventory, requirements and activity priority rating. Two example problems using fiscal year 1969 data are solved to illustrate the technique. The author recommends that a full scale test problem be constructed with an activity configuration which resembles that of the current ODP in order to better estimate the value of this approach to officer personnel planning.

TABLE OF CONTENTS

I. INTRODUCTION

The Bureau of Naval Personnel is confronted with the task of distributing approximately 75,000 officers, 70% of whom are members of the unrestricted line community. They thus have two major problems; that is, 1) to provide the personnel resources required and 2) to distribute these resources to meet requirements.

A third area of concern for the Bureau of Naval Personnel is the determination of manning levels. To put it another way, given a particular makeup of officer inventory and authorized billets (allowance), which billets must be left empty and which filled and by what grade and designator of officer. It is the CNO who is responsible for determining the allowance and making manpower authorizations. However since shortages exist in many categories of personnel with the result that the allowance cannot always be filled with the type of personnel required, it is BUPERS who is left with the development of manning levels (requirements).

The objective of this study was to develop an Officer Distribution Model for the unrestricted line community. The model was to distribute officer personnel by grade and designator to the various activities in the Navy in a most "feasible" manner. A "feasible" distribution would be to fit an officer by grade and designator to a billet with such requirements provided there is a need to fill that billet. If there is a billet which is unfilled and there is no perfect fit available but it has a greater need to be filled than another billet,

it should be filled if possible by up/down or by cross detailing. It is apparent that a "feasible" distribution is not necessarily optimal but it is a possible distribution under the existing constraints.

The model was to be compatible with the present Officer Distribution Plan (ODP) which is currently massaged by hand with the aid of reports prepared as an output of the Officer Management Simulation Model (OMSM). The model's format, activity configuration and officer categories were to be similar to the present ODP and it was to allow for cross and up/down detailing as does the ODP. The Officer Distribution Plan provides guidelines of requirements which the personnel planner (assignment officer or detailer) must meet while considering not only an officer's grade and designator but also his secondary qualifications such as past experience, p-codes, preference and data card. Approaches to detailing giving consideration to secondary qualifications have been outlined in theses by Johnson [1] and Daniels [2]. Both Johnson and Daniels addressed the problem of determining the optimal assignment of officers to the billet outlined in the ODP on a continuing time basis. Although both authors showed that their methods were feasible neither has been adopted by BUPERS.

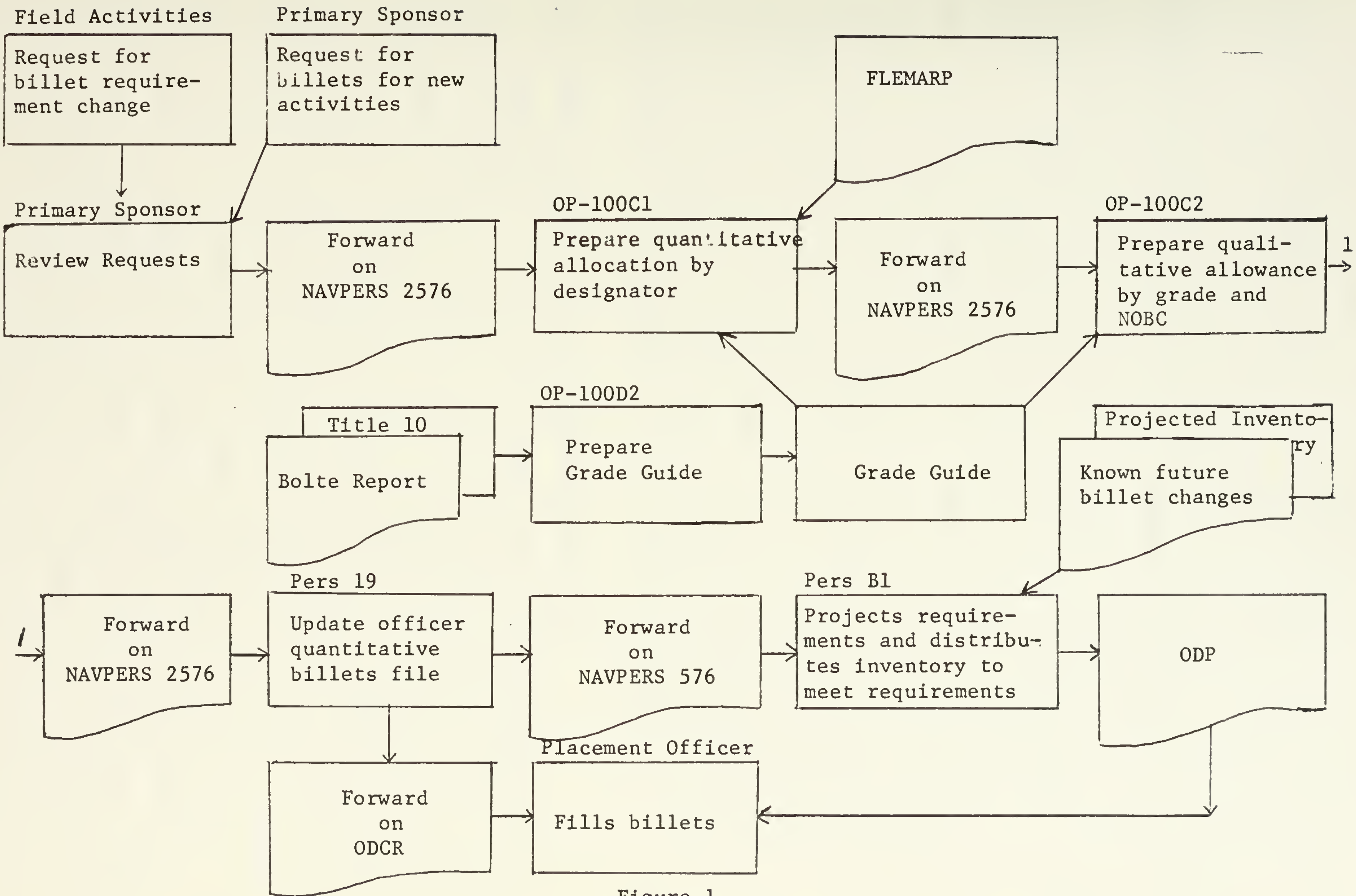
II. BACKGROUND

A. DEVELOPMENT OF AUTHORIZED BILLET REQUIREMENTS

Officer billet requirements are continually changing to meet the changing needs to the Navy. The required changes may be quantitative, qualitative, or both. These changes must be made within the constraints established by the Bolte Legislation Report, Stennis Ceiling, and Title 10 U.S. Code. A brief description of the current information flow, as shown in Figure 1, is presented below.

The information flow generally begins with requests for changes submitted by field activities to the primary sponsors; however, for new activities the request for initial billet designation is initiated by the primary sponsor. The sponsors review and forward the requests with recommendations to Deputy CNO (Personnel Plans Division), OP-10. Within OP-10 Division OP-100C1 reviews the requests and makes quantitative allocations by designator. The Officer Grade Plan, issued quarterly, and Fleet Manpower Allocation/Requirements Plan (FLEMARP), issued monthly, provide guides for these quantitative allocations within the official ceilings. The quantitative allocations by designators are forwarded to OP-100C2.

In OP-100C2, the qualitative allowance by grade and naval officer billet code (NOBC) are determined. The Officers Grade Guide, which is distributed quarterly to reflect changes in officer billets written, is used to maintain the proper grade structure within designators. This data is forwarded on a NAVPERS 2576 to BUPERS (Pers 19) to update the officer quantitative billets file.



Field Activities
Request for billet requirement change
Primary Sponsor
Request for billets for new activities
FLEMARP
Primary Sponsor
Review Requests
Forward on NAVPERS 2576
OP-100C1
Prepare quantitative allocation by designator
Forward on NAVPERS 2576
OP-100C2
Prepare qualitative allowance by grade and NOBC
1
Title 10
Bolte Report
OP-100D2
Prepare Grade Guide
Grade Guide
Projected Inventory
Known future billet changes
1
Forward on NAVPERS 2576
Pers 19
Update officer quantitative billets file
Forward on NAVPERS 576
Pers B1
Projects requirements and distributes inventory to meet requirements
ODP
Forward on ODCR
Placement Officer
Fills billets

Figure 1

The basic document which is affected by the officer billet changes is "Manpower Authorization" (NAVPERS 576) in which are listed quantitative and qualitative billets for each activity based upon the authorized strength of the Navy. This document along with known future billet changes and projected inventory are used by Pers Bl to develop the ODP. Pers 19 also compiles another report, the Officer Distribution Control Report (ODCR), which is an inventory-requirements match-by-activity on the basis of billet sequence code. This is used by the placement officer to determine the need to fill, 1) billets in which the currently assigned officer's projected rotation date is a minimum of six months in the future; 2) billets for new activities; and 3) new billets for an existing activity. It is at this point that the ODP is used as a guide in the distribution of personnel to the unfilled billets.

B. NEED FOR AN OFFICER DISTRIBUTION MODEL

The Navy is continually faced with the problem of an on-hand inventory of officers which is inadequate in either numbers, distribution or qualifications to meet the authorized strength of the Navy as established by the CNO in the "Manpower Authorization" for each activity. It is not feasible to change this strength authorization (allowance) even if the actual number of officers in the various grades fluctuates because the allowance is designed to represent the firm requirements to permit an activity to function close to peak efficiency and is thus a constant goal toward which to train and distribute personnel. It is highly probable that there will continue to

be an imbalance of officers by designator in the foreseeable future. Accordingly, an Officer Distribution Plan is needed to provide for the most equitable distribution of officers throughout the naval establishment.

Because of the continuous changes in officer inventory and in the authorized billet structure and the possibility of unexpected changes due to budget cuts, war, etc. it is necessary that an ODP be promulgated at least annually. In addition, because of the ever changing billet structure as reflected in the quarterly Officers Grade Guide, the ODP must be dynamic enough that it can quickly and easily reflect these changes. The current method of hand massaging the ODP takes approximately four months and, by the time it is complete, it no longer reflects the current inventory or billet structure.

In addition, the distribution sections of the OMSM, which is the only current existing attempt to mechanize the ODP, do not reflect the thinking which is put into the ODP. The OMSM fills all the sea billets first and then the remainder of the inventory is distributed to the shore activities. A certain percentage of both sea (in general over 100%) and shore billets are filled based on some vague priority system determined by placement officers performing the distribution.

III. THEORY OF THE MODEL

This section describes the officer personnel distribution system and shows that such a system may be generally represented by the linear programming model.

A. CHARACTERISTICS OF THE OFFICER PERSONNEL SYSTEM

Every officer in the U. S. Navy can be described in terms of rank and designator. For instance, we might accurately describe one individual as Commander, 11XX, surface or another individual as Lieutenant, 11XX, submarine. Since the above descriptions apply equally well to other officers, we have in effect defined subsets of the naval officer population. By proceeding to form all possible combinations of the three characteristics, we will exhaustively partition the naval officer population into a collection of mutually exclusive subsets.

Additional officer characteristics such as specialty (P-code), time in service, major areas of experience, and preference could also be considered in developing subsets. However, these will not be considered in developing the officer distribution model. The intent here is to develop a model which will distribute officers while only considering the major characteristics of rank and designator. This is the approach taken currently in the hand development of the ODP. The secondary characteristics are considerations which will be left for the placement officer to consider in trying to fill the billets as outlined in the ODP.

The subsets of billets to which the officers can be assigned have the same first two descriptors as the subsets of the officer population. The third description is the activity name. For instance, we might accurately describe one billet as Commander, 11XX, Guided Missile Destroyers or Lieutenant, 11XX, Sublant Staff. Consequently, all billets will fall within some such grouping forming the subsets of the billet population.

B. CLASSICAL PERSONNEL ASSIGNMENT PROBLEM

The personnel assignment problem we are concerned with is one in which there are m personnel categories with a_i individuals (officers) in category i. There are n job types (billet categories) and b_j individuals are needed in job type j. However, these job types can be in any number of h activities. This problem appears to be closely related to the assignment problems described in Hillier and Lieberman [3] and more generally in Hadley [4]. The objective of the classical assignment problem is usually to maximize total efficiency where r_{ij} is a measure of the efficiency of an individual from personnel category i working on job type j.

Mathematically, the model takes the form:

$$\text{Maximize } z = \sum_{j=1}^{n} \sum_{i=1}^{m} r_{ij} X_{ij} ,$$

$$\text{subject to} \quad \sum_{j=1}^{n} X_{ij} = a_i, \ i=1,\ldots,m,$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \ j=1,\ldots,n,$$

$$\text{and} \quad X_{ij} \geq 0, \text{ for all } i, j.$$

The classical assignment problem assumes that the number of individuals available is the same as the number of jobs to be filled, Σa_i, = Σb_j whereas in our problem $\Sigma a_i \neq \Sigma b_j$.

Such models have been used previously by other authors to represent personnel systems. Kossack and Beckwith have modeled a portion of the U. S. Air Force enlisted personnel system in 1959 [5]; and Charnes, Cooper and Stedry [6] have extended the use of the assignment model of linear programming to allow for dynamic interactions between personnel and positions.

Kossack and Beckwith's model was not implemented since they stated that there were deficiencies in the mathematics of the model. However, the results obtained from their project indicated that the development of such a personnel utilization model was entirely possible.

Charnes, Cooper and Stedry did not implement their model but showed the potential of linear programming as a possible way of developing alternatives to practices which assume only fixed job descriptions along with static organization charts and other arrangements that are prescribed almost independently of the available personnel.

IV. FORMULATION

A. CONSTRAINTS

As the Navy is constrained as to the size of the subsets making up the officer population, the first type of constraint of the model sets the sum of the unassigned and assigned i^{th} type officers equal to the total number of i^{th} type officers available for assignment. The number of i^{th} type officers available for assignment is the total number of i^{th} type officers, a_i, modified by the parameter θ_i to account for those who are in a transient, patient or prisoner (TP&P) status. Therefore, the first type of constraint is of the form

$$X_i + \sum_j \sum_k X_{ijk} = a_i \theta_i, \; 0 \leq \theta_i \leq 1, \; i=1,2,\ldots,m.$$

It is the object of the Navy to operate its activities at 100% of peacetime allowance or mobilization complements. However, shortages often exist in many categories of personnel; therefore, the allowance cannot always be filled with the type of personnel required, if at all. Also because of an increased tempo of operations in specific areas, it is often necessary to increase the personnel of involved activities above allowance at the expense of other activities. Consequently, there is a need to establish manning priorities by function which determine an upper bound on the number of officers to be assigned to the k^{th} activity. This leads to the second type of constraint which has the following form:

$$\sum_j \sum_i X_{ijk} \leq \sum_j p_k n_{jk}, \; 0 \leq p_k \leq 1.5, \; k = 1,2,\ldots,h.$$

The parameter p_k is an adjustment factor n_{jk}, the number of j^{th} type billets in the k^{th} activity specified by NAVPERS 576 and the OMSM.

The upper limit on the range of p_k is greater than one to allow for billet overfills. For example, overfills are necessary when work loads have increased due to war requiring day and night shifts or when special major projects have been requested from a particular office. In this model the upper limit has been set at 1.5 which should be sufficient to meet any situation and which is greater than any fill percentage given in Appendix A. The percentage manning of allowance, p_k, would have to be established by the CNO, under policy guidance from SECNAV.

A third type of constraint in the formulation of this assignment problem indicates which subsets of the officer population are eligible to fill a particular billet. The right hand side, $p_k n_{jk}$, is the number of officers desired to fill a particular billet. The third type of constraint has the form

$$\sum_{i \varepsilon A(j,k)} X_{ijk} + X_{jk} - Y_{jk} = p_k n_{jk}, \quad \begin{array}{l} j = 1,2,\ldots,n \;, \\ k = 1,2,\ldots,h \end{array}$$

where A (j,k) is the set of officers eligible to fill the j^{th} billet in the k^{th} activity. Because the situation can arise where there is insufficient inventory to fill billets to the desired level or conversely, there can be more inventory than necessary to fill billets to the desired level, non-negative slack variables, X_{jk} and Y_{jk}, were included in the constraint to maintain equality. The variable X_{jk} represents the number of j^{th} type billets in the k^{th} activity which are unfilled and the variable Y_{jk} represents the number of j^{th} type billets in

the k^{th} activity which are in excess of the minimum desired level. For example, this constraint could be:

Cdr.11XX + Lcdr.11XX + Unfilled Cdr.11XX + Overfilled Cdr.11XX
= Cdr.11XX destroyer billets.

The model is thus subject to three types of constraints: 1) size of the subsets within the officer population, 2) desired activity level of fill and, 3) billet eligibility. The third type of constraint is such that more officers can be assigned to a billet than is desired. However, the second type of constraint prevents an activity from being assigned more than its desired level of fill, which in effect limits the number of excess fills in the third type of constraint. If the situation arises such that the inventory is greater than needed to fill to the desired levels, it will be absorbed in the first type of constraint as unassigned personnel. If, on the other hand, there is insufficient inventory then the third type of constraint will indicate unfilled billets.

B. VALUES OF CONSTRAINT BOUNDS

Placement officers consider billets for assignment as much as six months prior to the actual reporting date. Consequently, they require an officer distribution plan which will guide them in filling the right billets with the proper officer type. In order to have a meaningful distribution plan for use in the future, projected values for a_i and n_{jk} are necessary for the period in which the ODP is to be used. Both of the projections are available from the OMSM.

C. OBJECTIVE FUNCTION

Initially an objective function was considered which would maximize an investment cost attached to the various subsets of officers such as developed by Daniels [2]. However, this was discounted primarily for two reasons. First, it would be difficult to equivocate between officers of different categories. For instance, by this method the worth of a Commander, jet pilot would be much greater than that of a Commander, destroyer type. Within their own specialties these costs would probably hold but when cross detailing to, say, a desk job they must have a more relative basis on which to be compared.

Second, the investment cost concept does not consider availability to the system. That is, given that a billet cannot be filled perfectly and that there are several other subsets of equally qualified officers eligible, it would normally be better to pick that subset which has the most excess inventory above that needed to fill its corresponding subsets of "perfect fit" billets. However, it might be better to direct a billet to be filled by an officer category which has less than 100% fill capability but which in turn can be filled by some other eligible officer category or be left unfilled if there is no need.

The objective of the model will be to maximize the utilization of the officers; that is to place the officers where they are most needed, downgrading some billets, not filling others. The intent is to fill the billets listed according to the major characteristics as best possible.

The objective function incorporated four aspects; billets which were filled, billets filled by up/down and cross detailing, unfilled and overfilled billets and unassigned officer inventory. Each of these aspects is considered in one or more of the three types of constraints.

A cost matrix was developed for each of the four aspects mentioned above. The concept is that of a "penalty cost" which imposes a cost for imperfect detailing, unfilled and overfilled billets, and unassigned personnel. A negative cost is assigned to perfect detailing. The resulting objective is therefore to minimize the cost.

Although the coefficients of the objective function variables will be referred to as "penalty cost," they are, in fact, measurements of relative values, ordinal measures[7].

The objective function as conceived will be to minimize

$$\sum_{\substack{i \\ i=j}} \sum_{j} \sum_{k} C^1_{ijk} X_{ijk} + \sum_{\substack{i \\ i \neq j}} \sum_{j} \sum_{k} C^2_{ijk} X_{ijk} + \sum_{j} \sum_{k} (C^3_{jk} X_{jk} +$$

$$C^4_{jk} Y_{jk}) + \sum_{i} C^5 X_i$$

where C^1_{ijk}, C^2_{ijk}, C^3_{jk}, C^4_{jk}, and C^5 are the penalty cost for perfect fills, imperfect fills, unfilled billets, overfilled billets and unassigned inventory, respectively. The variable X_{ijk} is the number of i^{th} type officers assigned to the j^{th} type billet in the k^{th} activity, X_{jk} is the number of unfilled j^{th} type billets in the k^{th} activity, Y_{jk} is the number of overfilled j^{th} type billets in the k^{th} activity and X_i is the number of unassigned i^{th} type officers.

Penalty costs need to be defined in such a manner that they can be used in the model to make good assignment decisions when several alternatives are available. The following three steps were considered as appropriate for making a good assignment:

1) Where possible, a correctly qualified officer will be assigned to a requirement.
2) When insufficient officers of a particular subset are available for all corresponding billets then highest priority activities will receive first consideration of inventory available.
3) When a billet cannot be perfectly filled then an attempt will be made to fill it from an eligible subset of officers which has an excessive inventory over requirement.

Cost associated with each of these steps can be levied on each assignment of an officer to a billet. By minimizing the magnitude of these costs we should maximize our planning effectiveness.

D. DERIVATION OF PENALTY COST

1. <u>Perfect Fill Penalty Cost</u>

When a perfect match is made between an officer and a billet it would seem that no penalty cost should be assessed. However, since it is not always desirable to fill all activities to exactly 100% level, it appears that there exists a priority ordering of billets. More specifically there appears to be a definite line for separation of activities into three

priorities; priority one corresponds to operational units, priority two to operating staffs and major shore facilities and priority three to training commands and miscellaneous shore facilities. Therefore, because of this ordering of activities, perfect fills should be assigned penalty cost which will result in assignments being made first to the highest priority activity and then to the others in descending priority order. To accomplish this, the set of C^1_{ijk} could be assigned arbitrary negative (values), C^1_1, C^1_2, C^1_3, the smallest corresponding to the highest priority activity, and largest to the lowest priority activity. For example, we might have C^1_{ijk} = {-3, -2, -1} where -3 is the penalty cost for priority one activities, -2 for priority two, and -1 for priority three.

2. Imperfect Fill Penalty Cost

For cross and up/down detailing the prime consideration was to assign from that subset of officers which had more inventory than necessary for the "perfect fit" billets it was required to fill. However, it can happen that all the subsets of officers eligible to fill a particular billet, A(j,k), have inventory shortages. Therefore, several other factors have to be considered such as the priority group of the activity under consideration, the priority group of the activity from whom a perfect fit would be taken and the "percentage availability" of the officer subsets in A(j,k).

The "percentage availability" form is calculated by taking the ratio of total inventory to "perfect fit" billets for each

subset of officers. This ratio, called f_i, can be written in equation form as

$$f_i = \frac{a_i}{\sum_k n_{ik}} \quad .$$

This ratio is calculated for all of the various officer types. Since the billet types, without regard to activity, correspond to the officer types, these ratios apply to the corresponding billet types. Therefore, the billet ratio, called f_j, is defined as

$$f_j = \frac{a_j}{\sum_k n_{jk}}$$

The percentage availability is equal to the billet ratio when i=j. By dividing each f_j by each f_i when $i \neq j$, a set of penalty cost $C_{ij} = \frac{f_j}{f_i}$ is developed. The elements C_{ii} are defined to be zero. A matrix, C, consisting of the C_{ij} elements can be constructed as follows:

$$
\begin{array}{ll|lllll}
 & & \multicolumn{5}{c}{\text{Billet Type}} \\
 & & 1 & 2 & 3 & \cdots & m \\
\hline
 & 1 & 0 & C_{12} & C_{13} & \cdots & C_{1m} \\
 & 2 & C_{21} & 0 & C_{23} & \cdots & C_{2m} \\
 & 3 & C_{31} & C_{32} & 0 & \cdots & C_{3m} \\
\text{Officer} & \cdot & \cdot & \cdot & \cdot & & \\
\text{Type} & \cdot & \cdot & \cdot & \cdot & & \\
 & \cdot & \cdot & \cdot & \cdot & & \\
 & m & C_{m1} & C_{m2} & C_{m3} & & 0
\end{array}
$$

Each C_{ij} element can be considered to be the penalty cost associated with assigning officer type i to billet type j. It assigns a high penalty cost when distributing an officer type with a low percentage availability, f_i, to a billet type with a high percentage availability, f_j, and a low cost when distributing a high percentage availability type to a billet with a low percentage availability.

The elements of the C matrix are next divided by the manning percentages for each priority group to create the imperfect fills penalty cost matrix. The elements are defined as follows:

$$C^2_{ijk} = \frac{C_{ij}}{p_k}$$

so that the penalty cost to fill a priority one billet is less than a priority two which in turn is less than a priority three. Thus, by knowing the priority groups to which an assignment of the same officer types is being considered, a choice can be made based on the C^2_{ijk} value.

3. Unfilled and Overfilled Billets Penalty Cost

In developing the penalty cost for unfilled billets it was assumed that choice of personnel was a function of an activity's priority; that is, highest priority should have first preference, next highest second preference, etc. Next, within a priority group it was assumed that a billet's order of importance is a direct function of its "perfect fit" officer. Thus a Cdr. billet is more important than a Lcdr. billet which in turn is more important than a Lt. billet, which in turn is

more important than a JG/Ens. billet. To establish this ranking a weight, d_j, is assigned to each billet grade. For example, we might have $d_j = \{100, 90, 80, 70\}$ where 100 is the weight for Cdr. billets, 90 for Lcdr. billets, 80 for Lt. billets and 70 for JG/Ens. billets. Thus C^3_{jk} would be a combination of d_j and p_k. In equation form:

$$C^3_{jk} = d_j p_k \quad .$$

Thus the cost will be higher for an unfilled high priority, high officer-grade billet than for an unfilled, low priority, low grade billet.

At times it might be necessary to overfill particular billets within an activity in order to attain the overall desired manning level. Thus the overfill variable, like the unfilled variable, will be a slack variable which is necessary to maintain equality within the third type of constraint. Since the purpose of this variable is only to maintain equality, the penalty cost, C^4_{jk}, assigned would probably be of no consequence to the outcome of most problems. In the example problems this penalty cost was set equal to zero. Increasing its value above zero would have the effect of directly penalizing for overfills. This might be more desirable in some problems than the indirect penalty costs imposed by having unfilled billets.

4. Unassigned Billets Penalty Cost

The necessity of having a penalty cost for an unassigned officer is to prevent the model from leaving officers unassigned when in fact they are required to meet desired

manning levels. This result did occur when the model was run without this penalty cost. This cost must be greater than the penalty cost for perfect fills and for cross detailing, that is $C^5 > \text{Max } C^2_{ijk}$, otherwise the model leaves personnel unassigned where it is less costly to do so. The following expression was used to determine C^5:

$$C^5 = \text{Max } C^2_{ijk} + 1 \ .$$

E. PERSONNEL DISTRIBUTION PROBLEM

The complete personnel distribution problem can be expressed as a linear program of the following form:

$$\text{Minimize} \sum_{\substack{i\\i=j}}\sum_{j}\sum_{k} C^1_{ijk} X_{ijk} + \sum_{\substack{i\\i\neq j}}\sum_{j}\sum_{k} C^2_{ijk} X_{ijk} + \sum_{j}\sum_{k} (C^3_{jk} X_{jk}$$

$$+ C^4_{jk} Y_{jk}) + \sum_{i} C^5 X_i$$

$$\text{Subject to} \quad X_i + \sum_i \sum_j \sum_k X_{ijk} = a_i \theta, \qquad i = 1,2,\ldots,m \ ,$$

$$\sum_j \sum_i X_{ijk} \leq \sum_j p_k n_{jk} \quad , \qquad k = 1,2,\ldots,h \ ,$$

$$\sum_{i \varepsilon A(j,k)} X_{ijk} + X_{jk} - Y_{jk} = p_k n_{jk}, \quad \begin{array}{l} j = 1,2,\ldots,n, \\ k = 1,2,\ldots,h \end{array}$$

and

$$Y_{jk} \geq 0, \ X_i \geq 0, \ X_{jk} \geq 0, \ X_{ijk} \geq 0 \text{ for all } i, j, k.$$

Such linear programs can be solved by the well-known Simplex method. Most computer facilities have this solution procedure as part of their standard software.

V. EXAMPLE PROBLEM

The following examples illustrate the formulation procedure and the solution results of the model. All computational phases of the problem were run on the IBM 360/67 computer at the Naval Postgraduate School.

The inputs to the problem were based on 1967 through 1969 data obtained from ODP and the Unrestricted Line and Associated Limited Duty Officer/Warrant Officer Manning Data of 6/30/69.

Officer and billet types were separated into five general groups as follows:

Commander (Cdr.)
Lieutenant Commander (Lcdr.)
Lieutenant (Lt.)
Lieutenant Junior Grade and Ensign (JG/ENS)
Warrant Officer (WO)

The first four of these groups were further separated by designator as follows:

11XX	Surface
11XX-s	Submarine
131X	
132X	
135X	
LDO	Surface
LDO-S	Submarine
LDO-A	Aviation

Warrant Officers were also separated as follows

Surface	(SWO)
Submarine	(SWO-S)
Aviation	(AWO)

The ODP is currently divided into 33 desks, each desk composed of similar type activities. The desks are further broken down by an activity mission code (AMC) which is a two

letter code that identifies an activity's mission. Consequently, all activities with a common mission, such as destroyers east cost, are uniquely identified. However, in this example the activity level of detail was not used. The "activities" of the example are the 33 desks.

The desks were classified into three groups: priority one, priority two, and priority three. The classification was based on the following general guidelines:

priority 1 - operation (ship and air squadrons)

priority 2 - operational staffs and major shore facilities

priority 3 - training commands and miscellaneous shore facilities

The desks are listed according to their priority grouping in Appendix A. The manning percentage developed for priority one was 103%, priority two, 98% and priority three, 89%. These were developed by dividing the total officers distributed to a priority group by the total authorized billets for that priority group using data given in the 1969 ODP. The data was limited to one year because previous years ODP's were configured differently.

The subsets of officers eligible for each billet must be evaluated and input into the model. For example, those made eligible for the billets listed under desk B1204 (Submarine and Staffs) are presented in the following matrix:

Officer Type

Billets	Cdr. 11XX-S	Lcdr. 11XX-s	Lcdr. LDO-S	Lt. 11XX-S	Lt. LDO-S	JG/ENS 11XX-S	JG/ENS LDO-S	SWO -S
Cdr. 11XX Sub.	1	1						
Lcdr. 11XX Sub.	1	1	1	1				
Lcdr LDO Sub.			1					
Lt. 11XX Sub.		1		1	1	1		
Lt. LDO Sub.								
JG/ENS 11XX Sub.				1	1	1	1	1
JG/ENS LDO Sub.							1	
SWO Sub.								1

Thus, in the case of billet type, Cdr. 11XX Sub., the billet can be filled by Cdr. 11XX-S, and Lcdr. 11XX-S as indicated by ones in the appropriate columns of the first row of the matrix. Consequently each row of the matrix, with the exclusion of the slack variables for unfilled and overfilled billets, corresponds to the coefficients of the left hand side of a constraint of the third type in the model.

A. TEST PROBLEM 1

The first test involved four specific desks with all the other desks combined into one larger desk. The desks used in the problem were:

Example Desk	Actual Desk Designation	Description	Priority Group
1	B1204	Submarines and Staffs	1
2	B1202a	Deslant	1
3	B1205	A6/Miscellaneous Jet	1
4	B1208	Washington	2
5		All others	3

The billet and invetory input data, given in Appendix B, the billet fill eligibility data, an example of which was given above, and the priority group manning percentages previously discussed make up the input data needed to formulate the model. A program was developed to convert this data into the proper format required by the IBM MPS/360 linear programming procedure. This program provides punch card output which can be directly used as the input for the MPS program.

The solution results are given in Appendix C and show a comparison by grade, designator and activity of the officer distribution as a result of the linear program, "L. P." column, the actual officer distribution as given in the 1969 ODP, "actual" column, and projected authorized billets as given in the 1969 ODP, "billets" column. In addition some results in the "L. P." column have alphabetic superscripts which are a cross reference indicating up/down and cross detailing. For example, superscript g indicates that the Cdr., 135X billet in activity 5 had a total of 68 officers detailed to it from two different officer subsets; one from Lcdr. 135X and 67 from Lcdr. 131X. In this specific case there were no perfect fills.

It will be noticed that in the linear program distribution the bulk of the up/down and cross detailing takes place in the lowest priority activity. Because there were sufficient

personnel in this case to meet the desired manning percentage for each activity, that is 103% for activities one, two and three, 98% for activity four and 89% for activity five; the following personnel were unassigned:

Grade	Designator	Quantity
JG/ENS	11XX	147
SWO	Surface	225
SWO	Aviation	143

At this point the desired manning percentages could be adjusted to accommodate this excess.

An inspection of the results in Appendix C reveals a reasonably close similarity in the distribution of officers by the linear program to the actual distribution given in the ODP. However, this should not be construed as implying that the detailers do not need the model. The speed at which the linear program accomplishes the distribution makes it a valuable planning guide for the detailers.

A similar test was conducted using the same input data with the exception of the perfect fill penalty costs. In the revised test C^1_{ijk} was set equal to -1.03, -.98 and -.89 for priority groups one, two and three respectively. This resulted in the low priority groups receiving "perfect fit" fills and high priority groups being filled by up/down and cross detailing when inventory shortages existed in an officer subset. For example, the Cdr. 135X billets in acitivities three and four had perfect fits in the original test problem. However, in the revised test these perfect fits were applied to activity five billets and were replaced by imperfect fits. A sensitivity

analysis of the C^1_{ijk} was performed on the test problem to determine at what point there would be a change in assignment to billets. The results of this analysis are shown in the following table:

Priority Group	Penalty Cost	Range
1	-3	-1.165 to -3.30
2	-2	-1.125 to -Infinity
3	-1	- .998 to -1.908

For example, if the perfect fit penalty cost for priority group one were increased higher than -1.165 while keeping the costs at -2 and -1 for the other groups respectively the Cdrs. 135X assigned to Cdr. 135X billets in activity three would be reassigned to activity five and the vacancies would be filled by Lcdr. 135X officer types. This verifies what happened in the revised test problem when -1.03 was used as the perfect fit penalty cost for priority group one.

The linear program gives part integer and part fraction assignment values. In the test problems any decimal values .5 or greater were rounded up and those less than .5 were rounded down. Because there were more than enough personnel to fill billets to the desired level in the first test the rounding off did not reduce the number of personnel designated for TP & P. In this problem the rounding off resulted in activity two receiving two less people and activity five three less people than the desired manning levels.

B. TEST PROBLEM 2

The second test conducted used the same data bank as the first test but desks were aggregated as follows:

Example Desk	Actual Desk Designation	Description	Priority Group
1	B1204	Submarines and Staff	1
2	B1202a,c,d	Destroyer and Mine Forces	1
3	B1205,a,b	Air Wings	1
4	B1210a, 1205c	Carriers	1
5	B1203a,c	Amphibious and Auxillary Forces	1
6	B1211	Vietnam	1
7		All desks in priority group two as listed in Appendix A	2
8		All desks in priority group three as listed in Appendix A	3

The apparent difference in looking at the data in this grouping is that there will not be a sufficient number of personnel to meet the total number of billets resulting from the desired manning level. Computationally this can be seen as follows:

Priority Group	Total Authorized Billets	Manning Percentages	Desired Manning Levels
1	24,229	103	24,956
2	13,236	98	12,971
3	10,566	89	9,304
	48,031		47,231

The actual available inventory was 47,261 x .94 = 44,425. Thus, after the inventory has been reduced by 6% for TP and P there is a shortage of 2,806 officers. The results of the linear programming solution are given in Appendix D. The resulting percentage of actual fill for each activity is as follows:

Activity	Fill Percentage
1	102
2	101
3	97
4	100
5	103
6	103
7	90
8	77

Overall priority group one had a 101% fill which is only 2% below the desired level. However, it can be seen that priority groups two and three took the brunt of the reduction, dropping 8% and 12% respectively below desired levels. Whether shortages should be absorbed somewhat evenly or absorbed mainly in the low priority activities as in this model is a matter of policy decision. If the distribution is not acceptable to the user then the fill percentages can be easily adjusted and the problem run again.

Rounding off in the second test resulted in TP and P being reduced by one person. Since the 6% figure used to determine the number in TP and P is only an estimate any small reduction in this category due to rounding off is insignificant.

The first example problem with four activities required approximately one minute of computer time including data card generation, input and output time while this problem required two minutes. The MPS program required approximately 400 iterations before an optimal solution was reached. It is impossible to say at this point how many iterations and how much time it would take to process a large number of actitivites. It is expected that the increase in computer time would be no worse than linear with the increase in the number of activities.

VI. DISCUSSION

In the derivation of the decision values and penalty cost, three significant assumptions were used. These are summarized as follows:

1) A billets order of importance is a direct function of its "perfect fit" officer grade;
2) there is an underlying priority structure among activities based on whether it is operational, staff, major shore or secondary shore activity;
3) percentage availability is the distinguishing factor among those designated as eligible to fill a billet.

It is realized that the first assumption does not recognize the outstanding, highly qualified officers of a grade who are obviously more capable than many in the next highest grade. The model does to some degree recognize this in that it allows for an officer to fill the next highest grade but only when there is a definite need. The second assumption was based strictly on subjective reasoning and requires much more thought before it can be incorporated into the model. It could also cause morale problems among those serving in the low priority group activities if their status became known. The third assumption results in the percentage availability of the officer subsets being constant over the entire range of assignments. Actually, the percentage availability would change with each additional assignment of an officer to a

billet if sequential assignment was done. However, this assumption is considered valid because the linear programming model assumes the entire range of assignments are made at a single instant in time.

A primary requirement of a linear programming formulation is that the objective function and every constraint must be linear in the variables. This requires that the measure of effectiveness and resource usage must be proportional to the level of each activity conducted individually. Since the penalty cost developed for perfect fills, unfilled and over-filled billets and unassigned personnel are strictly ranking measures and are not based on officer inventory or billets, they have been assumed to maintain the linearity of the objective function. Similarly, the imperfect fit penalty cost is based on the ratio of percentage availability of the officer subsets, f_i, to the billets, f_j, and is also a ranking measure. If only f_i had been used as the penalty cost the linearity would have been destroyed because

$$\frac{(X_{ijk} \cdot X_{ijk})}{n_{jk}}$$

would have appeared in the objective function.

The linear programming model was selected for use for several reasons. First, the linear programming model is readily solvable using the MPS Linear Programming package and can handle large numbers of constraints and variables. Second, the linear programming model can still be used when an integer solution is required. The integer programming phase of the MPS package

is still in the developmental stages, however. Third, once a solution has been found a parametric study or sensitivity analysis on any bound or penalty cost can be conducted quite easily using subroutines from the Linear Programming package.

The implementation of a linear programming distribution model, such as has been described in this thesis, can be expected to yield several advantages. First, because it is assumed that the ODP is a necessary management tool for efficient personnel assignment, the man-hours saved in the preparation of the ODP should be well worth the implementation effort. Second, the model could pinpoint problem areas, such as indicating billets where a shortage of officers exists, and the officer types needed to meet desired manning levels. Third, the model could be used for parametric analysis of personnel assignment policies used in the model. For example, what effect will changes in the manning percentages have on the officer distribution. Fourth, the model can be rerun frequently with little difficulty when changes occur in the system.

The most time-consuming phase of implementation would be the development of the special program to take the input data and merge it into the format required for input into the IBM MPS/360 Linear Programming procedure. The needed data are readily available from sources in the personnel system (a list is given in Appendix E). The special program, however, contains many aspects which require careful attention. For example, the designation of officers who are eligible for the

various billets in each activity, specification of the priority system and calculation of percentage-of-fill figures for each priority group are required. In the model development and implementation of a real world problem officer categories and activities should be defined in a way that is meaningful to the Navy system; however, these categories should be kept to a minimum while still considering homogeniety. It is desirable to keep the basic categories as inclusive as possible because the time and effort required to compute a solution is related to the number of states considered. Further, if a full problem were run using the current IBM MPS/360 Linear Programming package, only 4999 constraints could be handled. Thus if an AMC is used in defining an activity as in the current ODP 360 codes would result and the constraints per activity would have to be limited to 14. In the two test problems, constraints averaged 22 per activity which would mean an excess of nearly 2900 constraints if AMC defined an activity.

Although not tested in this model a possible approach for reducing the volume of constraints is proposed. Rather than running the problem with a constraint equation for each billet subset within an activity several of these constraints could be combined and limited by the desired manning level for portions of the activity. This would, in the extreme case, reduce the number of constraints per activity to two, an activity constraint and a personnel constraint. For example suppose an activity consists of three different billets with

a total desired manning level of 100. Under current procedures, the constraints are as follows:

$$\sum_{i\varepsilon A(1,k)} X_{i1k} + X_{1k} - Y_{1k} = 20 \qquad \text{(E1)}$$

$$\sum_{i\varepsilon A(2,k)} X_{i2k} + X_{2k} - Y_{2k} = 30 \qquad \text{(E2)}$$

$$\sum_{i\varepsilon A(3,k)} X_{i3k} + X_{3k} - Y_{3k} = 50 \qquad \text{(E3)}$$

The proposed constraint would be

$$.2\ E(1) + .3\ E(2) + .5\ E(3) \leq 100 \ .$$

This would be similar to combining the second and third constraints of the previously developed model. Resolution of assignments within each billet subset would then have to be made by hand.

Although the MPS Linear Programming Package does not give integer results required in assignment problems it can be easily adjusted by rounding up for decimal values greater than or equal to .5 and rounding down for decimal values less than .5. It was shown in the example problems that rounding off will have little effect on the results except for possibly a slight reduction in TP&P which itself is an estimation. Of course the penalty cost may substantially increase. If a better integer solution is desired one could wait until the MPS Integer Linear Programming package is available or a special program such as Gomery's cutting plane method [8] could be used.

VII. SUMMARY AND CONCLUSIONS

The problem addressed was, "Given a set of officer resources and requirements, how should they be distributed to 'best' meet the needs of the Navy?". The outgrowth of attempting to answer this question was a new methodology for developing the Officer Distribution Plan (ODP).

The initial analysis of the officer distribution problem indicated that it resembled the classical personnel assignment problem of linear programming. One major difference was that the numbers of resources and the number of requirements would not match within the various billet categories.

The constraint set consisted of constraints which limit the number of officers within each labor category, the number of billets per activity, and the officers eligible to fill particular billets.

To provide an efficient allocation of personnel to billets a measure of assignment effectiveness was developed based on availability of resources, ordering of grades and ordering of activities. The measure took the form of penalty costs. The minimum penalty costs were found to occur when an officer was assigned to a billet which required exactly his characteristics. The penalty costs were developed so that minimizing total penalty costs would correspond to maximizing assignment effectiveness.

An example was presented to demonstrate the functioning of the model. The data used in this example was taken from actual FY '69 unrestricted line data.

It appears that an officer personnel distribution plan can be evolved in a quick, effective and timely manner with this model. With such an updated version of the ODP, detailers will be able to plan more efficiently. Also, since this model readily adapts itself to parametric studies, it can be used to aid in policy decisions including where should cutbacks be made when required or how does a reduction in certain officer categories affect the entire system.

APPENDIX A

PRIORITY GROUPINGS

Priority 1

Desk	Activity	FY '69 ODP Distribution	Billets	% Fill
1201a	Carriers	2522	2189	115
1202a	Destoryers, Atlantic	2925	2666	110
1202c	Destroyers, Pacific	2825	2799	101
1202d	DE/Mine Forces	1597	1395	114
1203a	Amphibious Ships	2444	2171	113
1203c	Auxillary Ships	1853	1735	107
1204	Submarine/Staffs	3422	3402	101
1205	A6	1465	1649	89
1205a	A4/A7	1017	1092	93
1205b	VF/VFP	1192	1397	85
1205c	Helo/CV Prop	2500	2634	95
1211	Vietnam	1214	1190	102
	Average % fill			103

Priority 2

Desk	Activity	FY '69 ODP Distribution	Billets	% Fill
1201b	Aviation/Fleet Staffs	1704	1713	99
1203b	Aux./Amp./MSTS Staffs	1454	1394	104
1205d	Shore Prop.	2676	2790	96
1208	Washington	821	918	90
1209	Joint/Navy Staffs	1248	1338	93
1209a	Recruiting	1246	1205	103
1212	CEC	63	65	97
1213	Material Command	944	896	105
1214	Air Systems Cmds.	1161	1202	97
1216	National Security	380	418	91
1217	Communications	1235	1297	95
	Average % fill			98

APPENDIX A

Priority 3

Desk	Activity	FY '69 ODP Distribution	Billets	% Fill
1207	Naval Air Stations	1575	1723	88
1207a	Aviation Training	2475	2824	88
1210	Intelligence	642	729	88
1215	Supply and Finance	141	157	90
1218	Naval Researve Units	604	720	84
1218a	NROTC	449	470	96
1219	Metro and Oceano.	205	211	97
1220	Medical	2	4	50
1222	Ordnance System Cmd.	1026	1155	89
1223	School Staffs	2334	2573	91
	Average % fill			89

APPENDIX B

INVENTORY AND BILLET DATA

Test 1

		1	2	3	4	5	Total Billets	Available Inventory
Cdr	11XX		184		311	2153	2648	1808
	11XX-S	259				41	300	330
	131X		1	87	227	2094	2409	2729
	132X			1	2	145	148	50
	135X			3	2	76	81	77
	LDO					24	24	127
	LDO-S							
	LDO-A			2		8	10	64
Lcdr	11XX		247		182	3570	3999	2569
	11XX-S	559				41	600	570
	131X		8	224	80	2829	3141	3178
	132X		1	55	2	802	860	381
	135X			60	1	123	184	218
	LDO		13		1	218	232	1099
	LDO-S	13				7	20	40
	LDO-A			7		246	253	698

APPENDIX B

Test 1 (Cont.)

		1	2	3	4	5	Total Billets	Available Inventory
Lt	11XX		615		60	4905	5580	4024
	11XX-S	1327				23	1350	1360
	131X		7	515	23	4826	5371	3098
	132X			207		1572	1779	1486
	135X		2	27	2	151	182	356
	LDO		49		1	1106	1156	1801
	LDO-S	161				14	175	310
	LDO-A			61		540	601	883
JG/ ENS	11XX		1184		14	6284	7482	10860
	11XX-S	851				49	900	950
	131X			77	2	2769	2848	2334
	132X		1	177		1156	1334	913
	135X		4	47		335	386	1240
	LDO		75			399	474	93
	LDO-S	69				11	80	40
	LDO-A			12		226	238	47
SWO			275		8	2012	2295	2454
SWO-S		163				37	200	255
SWO-A				87		604	691	819
Totals		3402	2666	1649	918	39396	48031	47261

APPENDIX B

INVENTORY AND BILLET DATA

Test 2

		1	2	3	4	5	6	7	8	Total Billets	Available Inventory
Cdr	11XX		405		43	114	77	1308	701	2648	1808
	11XX-S	259							41	300	330
	131X		2	255	348	30	10	1092	672	2409	2729
	132X			1	1	3		37	106	148	50
	135X			3	23		2	21	32	81	77
	LDO							13	11	24	127
	LDO-S										
	LDO-A			2		1		7		10	64
Lcdr	11XX		640		64	273	131	1590	1301	3999	2569
	11XX-S	559							41	600	570
	131X		14	562	481	12	15	1188	869	3141	3178
	132X		2	88	82			145	543	860	381
	135X			62	22	3	3	70	24	184	218
	LDO		23		3	13	7	110	76	232	1099
	LDO-S	13							7	20	40
	LDO-A			25	39		1	70	118	253	698

APPENDIX B

Test 2 (Cont.)

		1	2	3	4	5	6	7	8	Total Billets	Available Inventory
Lt	11XX		1532		133	776	322	1669	1158	5580	4024
	11XX-S	1327							23	1350	1360
	131X		7	963	941	31	24	1135	2270	5371	3098
	132X		1	358	283		1	529	607	1779	1486
	135X		2	47	24		15	87	7	182	356
	LDO		110		156	165	84	357	284	1156	1801
	LDO-S	161							14	175	310
	LDO-A		1	70	119		9	154	248	601	883
JG/ ENS	11XX		3430		546	1720	407	1145	234	7482	10860
	11XX-S	851							49	900	950
	131X			791	982	18	2	831	224	2848	2334
	132X		1	355	24		1	454	499	1334	913
	135X		4	132	44	18	8	81	99	386	1240
	LDO		151		26	24	28	212	33	474	93
	LDO-S	69							11	80	40
	LDO-A			93	25		6	71	43	238	147
SWO			535	1	250	715	34	698	62	2295	2454
SWO-S		163							37	200	255
SWO-A				240	164		3	162	122	691	819
Totals		3402	6860	4048	4823	3906	1190	13236	10566	48031	47361

APPENDIX C

PROBLEM RESULTS TEST 1

		Activities 1 LP	1 Actual	1 Billets	2 LP	2 Actual	2 Billets	3 LP	3 Actual	3 Billets
Cdr	11XX				190	177	184			
	11XX-S	267	324	259						
	131X				1		1	90	122	87
	132X							1	1	1
	135X							3	1	3
	LDO					1				
	LDO-A							2	1	2
Lcdr	11XX				254	210	247			
	11XX-S	576 [a]	506	559						
	131X				8	7	8	231	250	224
	132X				1	1	1	57	51	55
	135X							62	6	60
	LDO				13	35	13			
	LDO-S	13	36	13						
	LDO-A							7	17	7

APPENDIX C

PROBLEM RESULTS TEST 1

Activities (Cont)

		4 LP	4 Actual	4 Billets	5 LP	5 Actual	5 Billets
Cdr	11XX	305	209	311	1915[d]	1422	2153
	11XX-S				36	6	41
	131X	222	210	227	1864[e]	2397	2094
	132X	2	4	2	129[f]	28	145
	135X	2	4	2	68[g]	72	76
	LDO		4		21	122	24
	LDO-A				7	63	8
Lcdr	11XX	178	145	182	3177[h]	2214	3570
	11XX-S				36[j]	64	41
	131X	78	53	80	2518	2868	2829
	132X	2	8	2	714[k]	321	802
	135X	1	10	1	110[m]	202	81
	LDO	1	12	1	194	1052	218
	LDO-S				6	4	7
	LDO-A		3		219	678	246

APPENDIX C

PROBLEM RESULTS TEST 1

Activities (Cont)

		1			2			3		
		LP	Actual	Billets	LP	Actual	Billets	LP	Actual	Billets
Lt	11XX				633	543	615			
	11XX-S	1366[b]	1358	1327						
	131X				7		7	530	321	515
	132X					2		213	148	207
	135X				2		2	28	32	27
	LDO				50	59	49			
	LDO-S	166	307	161						
	LDO-A					2		63	97	61
JG/ENS	11XX				1220	1687	1184		21	
	11XX-S	877	626	851						
	131X							79	60	77
	132X				1	1	1	182	181	177
	135X				4		4	48	68	47
	LDO				77	5	75			
	LDO-S	38	14	69						
	LDO-A							12[c]	15	12
SWO					283	195	275			
SWO-S		201	251	163						
AWO								90	73	87
Totals		3504	3422	3402	2744	2925	2666	1698	1465	1649

APPENDIX C

PROBLEM RESULTS TEST 1

Activities (Cont)

		4 LP	4 Actual	4 Billets	5 LP	5 Actual	5 Billets
Lt	11XX	59	65	60	4365[n]	3416	4905
	11XX-S				21[o]	2	23
	131X	23[u]	11	23	4294[p]	2766	4826
	132X		8		1399[q]	1328	1572
	135X	2	4	2	135	320	151
	LDO	1	17	1	984	1722	1106
	LDO-S				12	3	14
	LDO-A		1		481	783	540
JG/ ENS	11XX	14	40	14	6099	9212	7022
	11XX-S				43[r]	324	49
	131X	2		2	2464[s]	2274	2769
	132X				1029[t]	550	1333
	135X		2		298	1170	35
	LDO				10	88	399
	LDO-S					26	11
	LDO-A				44	32	226
SWO		8	10	8	1791	2249	1212
SWO-S					38	4	37
AWO					538	745	904
Totals		900	821	918	35059	38527	39292

APPENDIX C

UP/DOWN AND CROSS DETAILING CROSS REFERENCE LIST-TEST 1

Code	Qty.	Officer Type	Qty.	Officer Type	Qty.	Officer Type
a	22	Lt. 11XX-S	18	Lcdr. LDO-S		
b	110	Lt. LDO-S				
c	12	Lt. LDO-A				
d	67	Cdr. 135X	98	Cdr. LDO	52	Lcdr. 11XX
	486	Cdr. 13XX	7	Cdr. 11XX-5		
e	47	Lcdr. 135X	51	Cdr. LDO-A		
f	85	Lcdr. 131X				
g	1	Lcdr. 135X	67	Lcdr. 131X		
h	459	Lt. 11XX	788	Lcdr. LDO		
j	36	Lcdr. LDO				
k	415	Lcdr. LDO-A				
m	15	Lcdr. LDO-A				
n	1095	JG/ENS 11XX	639	Lt. LDO		
o	3	Lt. LDO-S	18	Lt. LDO		
p	168	Lt. 135X	274	Lt. LDO-A	879	JG/ENS 131X
	599	JG/ENS 135X				
q	215	JG/ENS 135X				
r	27	JG/ENS 11XX				
s	1230	JG/ENS 11XX				
t	354	JG/ENS 11XX				
u	23	JG/ENS 11XX				

APPENDIX D

PROBLEM RESULTS TEST 2

Activities

		1 LP	1 Actual	1 Billets	2 LP	2 Actual	2 Billets	3 LP	3 Actual	3 Billets
Cdr	11XX				417	389	405			
	11XX-S	267	324	259						
	131X				2	2	2	263	292	255
	132X							1	1	1
	135X							3	1	3
	LDO					2				
	LDO-A							2	1	2
Lcdr	11XX				659	543	640			
	11XX-S	576[a]	506	559						
	131X				14	9	14	579	546	562
	132X				2	4	2	91	92	88
	135X							64	23	62
	LDO				24	73	23			
	LDO-S	13	36	13						
	LDO-A							26	39	25

APPENDIX D

PROBLEM RESULTS TEST 2

Activities (Cont)

		4			5			6		
		LP	Actual	Billets	LP	Actual	Billets	LP	Actual	Billets
Cdr	11XX	44	16	43	117	104	114	79	73	77
	11XX-S									
	131X	358	338	348	31	39	30	10	10	10
	132X	1	14	1	3	3	3			
	135X	24	10	23				2	2	2
	LDO		5			1			2	
	LDO-A		11		1	1	1			
Lcdr	11XX	66	29	64	281	226	273	135	119	131
	11XX-S									
	131X	495	594	481	12	27	12	15	18	15
	132X	84	63	82					1	
	135X	23	43	22	3	3	3	3	3	3
	LDO	3	71	3	13	69	13	7	20	7
	LDO-S									
	LDO-A	40	49	39				1	1	1

APPENDIX D

PROBLEM RESULTS TEST 2

Activities (Cont)

		7			8		
		LP	Actual	Billets	LP	Actual	Billets
Cdr	11XX	1281[g]	872	1308	624[p]	354	701
	11XX-S				37	6	41
	131X	1070	1201	1092	598	847	672
	132X	36	20	37	95[q]	12	106
	135X	21	38	21	29[r]	26	32
	LDO	13	97	13	10	20	11
	LDO-A	7	23	7		28	
Lcdr	11XX	1559[h]	985	1590	1158[s]	667	1301
	11XX-S				36[t]	64	41
	131X	1164	917	1188	774[u]	1067	869
	132X	142	211	145	483[v]	10	543
	135X	69	108	70	21	38	24
	LDO	108	297	110	68	569	76
	LDO-S				6	4	7
	LDO-A	69	106	70	105	503	118

APPENDIX D

PROBLEM RESULTS TEST 2

Activities (Cont)

		1			2			3		
		LP	Actual	Billets	LP	Actual	Billets	LP	Actual	Billets
Lt	11XX				1578[c]	1369	1532			
	11XX-S	1366[b]	1358	1327						
	131X				7		7	992	711	963
	132X				1	2	1	369	288	358
	135X				2		2	48	56	47
	LDO				113	160	110			
	LDO-S	166	307	161						
	LDO-A				1	5	1	72	196	70
JG/ ENS	11XX				3533	4379	3430		25	
	11XX-S	877	626	851						
	131X							815	687	791
	132X				1	1	1	366	329	355
	135X				4		4	136	184	132
	LDO				34	7	151			
	LDO-S	38	14	69						
	LDO-A							96[d]	46	93
SWO					551	402	535	1		1
SWO-S		168	251	163						
AWO									157	240
Totals		3471	3422	3402	6943	7347	6860	3924	3674	4048

APPENDIX D

PROBLEM RESULTS TEST 2

Activities (Cont)

		4			5			6		
		LP	Actual	Billets	LP	Actual	Billets	LP	Actual	Billets
Lt	11XX	137	116	133	789	502	766	332[f]	339	322
	11XX-S									
	131X	969	717	941	32	9	31	25	6	24
	132X	291	154	283		2		1	6	1
	135X	25	47	24		1		15	15	15
	LDO	161	138	156	170	269	165	87	101	84
	LDO-S									
	LDO-A	123	199	119		17		9	9	9
JG/ ENS	11XX	562	873	546	1772	2385	1720	419	415	407
	11XX-S									
	131X	1011	837	982	19		18	2	11	2
	132X	25	176	24				1	1	1
	135X	45	146	44	19	19	18	8	8	8
	LDO		5	26	25	2	24	29		28
	LDO-S									
	LDO-A	26[e]	18	25		1		6	6	6
SWO		258	233	250	736	612	715	35	45	34
SWO-S										
AWO		70	120	164		5		3	3	3
Totals		4841	5022	4823	4023	4297	3906	1224	1214	1190

APPENDIX D

PROBLEM RESULTS TEST 2

Activities (Cont)

		7			8		
		LP	Actual	Billets	LP	Actual	Billets
Lt	11XX	1636[j]	1185	1669	1030[w]	513	1158
	11XX-S				20[z]	2	23
	131X	1112[k]	774	1135	2021[x]	881	2270
	132X	518	547	529	540[y]	487	607
	135X	85	124	87	6	113	7
	LDO	350	951	357	253	182	284
	LDO-S				12	3	14
	LDO-A	151	312	154	221	145	248
JG/ ENS	11XX	1123[m]	1987	1145		896	234
	11XX-S					324	49
	131X	815[n]	627	831		172	224
	132X	445[o]	397	454		9	449
	135X	79	401	81		482	99
	LDO		4	212		75	33
	LDO-S					26	11
	LDO-A		36	71		40	43
SWO			556	698		606	62
SWO-S						4	37
AWO			156	162		278	122
Totals		11853	12932	13236	8147	9453	10566

APPENDIX D

UP/DOWN AND CROSS DETAILING CROSS REFERENCE LIST-TEST 2

Code	Qty.	Officer Type	Qty.	Officer Type	Qty.	Officer Type
a	32	Lcdr. 13XX	18	Lcdr. LDO-S		
b	113	Lt. LDO-S	16	JG/ENS 11XX-S		
c	577	JG/ENS 11XX				
d	52	Lt. LDO-A				
e	26	Lt. LDO-A				
f	332	JG/ENS 11XX				
g	240	Lcdr. 11XX				
h	813	Lt. 11XX				
j	1636	JG/ENS 11XX				
k	225	JG/ENS 131X				
m	72	SWO-S	726	SWO		
n	252	AWO	563	JG/ENS 11XX		
o	445	AWO				
p	7	Cdr. 11XX-S	233	Cdr. 13XX	97	Cdr. LDO
	287	Lcdr. 11XX				
q	22	Lt. 11XX-S	45	Cdr. LDO-A	22	Lcdr. 135X
r	6	Cdr. LDO-A				
s	365	Lt. 11XX	19	Lt. 11XX-S	774	Lcdr. LDO
t	36	Lcdr. LDO				
u	89	Lcdr. LDO-A				
v	327	Lcdr. LDO-A	117	Lt. 135X		

APPENDIX D

UP/DOWN AND CROSS DETAILING
CROSS REFERENCE LIST-TEST 2 (Cont.)

Code	Qty.	Officer Type	Qty.	Officer Type	Qty.	Officer Type
w	539	Lt. LDO	491	JG/ENS 11XX		
x	347	JG/ENS 131X	241	JG/ENS 131X	550	JG/ENS 135X
	678	Lt. 11XX	35	Lt. 135X	170	Lt. LDO-A
y	324	JG/ENS 135X				
z	20	Lt. LDO				

APPENDIX E

DATA INFORMATION

Data	Origin	Comments
Authorized Billets	"Manpower Authorization" (NAVPERS 576)	quantitative and qualitative listing of billets for each activity
Projected Billets	OMSM	display by community, AMC and rank
Projected Inventory	OMSM	projects inventory by rank, year group and designator
Billet Eligibility		requires development
Priority Groups		requires development
Manning Percentages		requires development

```
THE PURPOSE OF THIS PROGRAM IS TO TAKE A LARGE VOLUMN OF DATA OF
FIVE TYPES,PROJECTED BILLETS,PROJECTED INVENTORY ,FILL PERCENTAGES,
GRADE RANKINGS(DELTA),AND BILLET ELLIGIBILITY(AMATX),AND DEVELOPE
THE PENALTY COSTS AND BOUNDS OF THE LINEAR PROGRAMMING MODEL AND
MERGE THIS INTO THE PROPER FORMAT FOR INPUT AS DATA INTO THE MPS
PROGRAM. THE OUTPUT OF THIS PROGRAM IS IN THE FORM OF PUNCH CARDS.

      INTEGER AMATX(45,225),B(225),OFFTY,NOACT,ACT(250),COLMN(250),
     1BETA(250),RANK(250),ROW(250),N(45),BILLS(45, 5),I,J,K,L,M,O,P,R
      REAL ACTBN(5),COST(45,45),PENCS1,OFBND(45),FRATE(45),PENCS(250),
     1DELTA(45),THETA,COSTO,SBILLS(45),INV(45),KBILL(45),PRI(5),
     2PENCS2,PENCS3,PRI1,PRI2,PRI3,BILBN(250)
      NOACT=5
      NOAMA=225
      O=0.0
      PRI1=1.03
      PRI2=0.98
      PRI3=0.89
      PENCS1=-3.0
      PENCS2=-2.0
      PENCS3=-1.0
      THETA=0.0

INPUT DATA

      DO 1 K=1,NOACT
    1 READ(5,30) (BILLS(J,K),J=1,45)
      READ(5,10)(PRI(K),K=1,NOACT)
      READ(5,20)(((AMATX(I,P),I=1,45),B(P)),P=1,NOAMA)
      READ(5,37)(INV(I),I=1,45)
      READ(5,36)(DELTA(I),I=1,45)

CALCULATE AND PUNCH CARDS IN FORMAT OF ROWS SECTION OF MPS DATA INPUT
SUCH THAT THE HEADING IS FOLLOWED BY ACTIVITY NAMES,OFFICER TYPES,
AND BILLET ELIGIBILITY ROW NAMES WHICH MAKE UP THE MODEL CONSTRAINTS.

      WRITE (6,11)
      WRITE (6,21)
      WRITE (6,31)
      DO 500 K=1,NOACT
      WRITE(6,43) K
  500 CONTINUE
      DO 600 I=1,45
      O=I+10
      WRITE (6,51) O
  600 CONTINUE
      P=0
```

```
      DO 601 K=1,NOACT
      DO 602 J=1,45
      P=P+1
      DO 603 I=1,45
      IF(AMATX(I,P).NE.1) GO TO 603
      WRITE(6,55) P
      GO TO 602
  603 CONTINUE
  602 CONTINUE
  601 CONTINUE

   CALCULATE AND PUNCH CARDS IN FORMAT OF COLUMNS SECTION OF MPS DATA
   DEVELOPE C(I,J) MATRIX FOR CROSS AND UP/DOWN DETAILING
   DETERMINE IF A PERFECT FILL(I=J).IF SO ASSIGN APPROPRIATE PENALTY
   COST(PENCS1,PENCS2,PENCS3).IF NOT ASSIGN APPROPRIATE PENALTY COST
   FOR UP/DOWN CROSS DETAILING(PENCS).

      DO 100 J=1,45
      SBILLS(J)=0.0
      DO 101 K=1,NOACT
  101 SBILLS(J)=SBILLS(J)+BILLS(J,K)
  100 CONTINUE
      DO 102 J=1,45
      FRATE(J)=0.0
      IF(SBILLS(J).EQ.0.0) GO TO 102
      IF(INV(J).EQ.0.0) GO TO 102
      FRATE(J)=INV(J)/SBILLS(J)
  102 CONTINUE
      DO 200 I=1,45
      DO 201 J=1,45
      COST(I,J)=0.0
      IF (I.EQ.J) GO TO 39
      IF(FRATE(I).NE.0.0) GO TO 4
      COST(I,J)=0.0
      GO TO 201
    4 IF(FRATE(J).NE.0.0) GO TO 5
      COST(I,J)=0.0
      GO TO 201
    5 COST(I,J)=FRATE(J)/FRATE(I)
      GO TO 201
   39 COST(I,J)=0.0
  201 CONTINUE
  200 CONTINUE
      WRITE (6,71)

   CALCULATE PENALTY COST FOR PERFECT FILLS AND UP/DOWN DETAILING

      DO 700 I=1,45
```

```
      L=0
      P=0
      DO 701 K=1,NOACT
      DO 702 J=1,45
      P=P+1
      IF(AMATX(I,P).NE.1) GO TO 702
      L=L+1
      IF(I.EQ.J) GO TO 49
      IF(COST(I,J).NE.0.0) GO TO 73
      PENCS(L)=0.0
      GO TO 72
   49 IF(PRI(K).EQ.PRI1) GO TO 40
      IF(PRI(K).EQ.PRI2) GO TO 41
      IF(PRI(K).EQ.PRI3) GO TO 42
   40 PENCS(L)=PENCS1
      GO TO 74
   41 PENCS(L)=PENCS2
      GO TO 74
   42 PENCS(L)=PENCS3
      GO TO 74
   73 PENCS(L)=COST(I,J)/PRI(K)

   CALCULATE PENALTY COST (THETA) FOR UNASSIGNED INVENTORY

   74 IF(PENCS(L).GE.THETA) THETA=PENCS(L)
   72 COLMN(L)=(1000000.0)*(I+10)+(10000.0)*(J+10)+K
      ACT(L)=K
      RANK(L)=I+10
      ROW(L)=P
      BETA(L)=B(P)
  702 CONTINUE
  701 CONTINUE
      IF(L.EQ.0) GO TO 700
      DO 800 M=1,L
      WRITE (6,81) COLMN(M),PENCS(M),ACT(M),COLMN(M),RANK(M),ROW(M)
  800 CONTINUE
  700 CONTINUE

   CALCULATE PENALTY COST FOR UNFILLED BILLETS

      ROW(L)=C
      L=0
      P=0
      PENCS(L)=0.0
      COLMN(L)=0
      DO 900 K=1,NOACT
      DO 901 J=1,45
      P=P+1
```

```
      DO 902 I=1,45
      IF(AMATX(I,P).NE.1) GO TO 902
      L=L+1
      COLMN(L)=(10000.0)*(J+10)+K
      PENCS(L)=DELTA(J)*PRI(K)*100.
      ROW(L)=P
      GO TO 901
  902 CONTINUE
  901 CONTINUE
  900 CONTINUE
      DO 903 M=1,L
  903 WRITE(6,91) COLMN(M),PENCS(M),ROW(M)
      DO 904 M=1,L
  904 WRITE(6,96) COLMN(M),ROW(M)
      THETA=THETA+1.0
      DO 935 I=1,45
      N(I)=I+10
  935 WRITE(6,97) I,THETA,N(I)

 CALCULATE AND PUNCH CARDS IN FORMAT OF RHS SECTION OF MPS DATA INPUT

      WRITE (6,92)
      IF(NOACT.EQ.(NOACT/2)*2) GO TO 89
      NOACT=NOACT-1
      WRITE(6,93)(( K   ,ACTBN(K)),K=1,NOACT)
      NOACT=NOACT+1
      WRITE(6,98)NOACT,ACTBN(NOACT)
      GO TO 79
   89 WRITE(6,93)(( K   ,ACTBN(K)),K=1,NOACT)

 CALCULATE BOUNDS(ACTBN) ON NO. OF BILLETS FOR KTH ACTIVITY

   79 DO 300 K=1,NOACT
      KBILL(K)=0.0
      DO 301 J=1,45
  301 KBILL(K)=KBILL(K)+BILLS(J,K)
      ACTBN(K)=PRI(K)*KBILL(K)
  300 CONTINUE
      DO 750 I=1,45

 CALCULATE BOUNDS ON NUMBER OF ITH TYPE OFFICERS(OFBND)

      OFBND(I)=0.0
      OFBND(I)=INV(I)*(.94)
  750 N(I)=I+10
      OFFTY=45
      IF(OFFTY.EQ.(OFFTY/2)*2) GO TO 99
      OFFTY=OFFTY-1
```

```
      WRITE(6,94)((N(I),OFBND(I)),I=1,OFFTY)
      OFFTY=OFFTY+1
      WRITE(6,88)OFFTY,OFBND(OFFTY)
      GO TO 69
   99 WRITE(6,94)((N(I),OFBND(I)),I=1,OFFTY)

C     CALCULATE BOUNDS ON NUMBER OF JTH TYPE BILLETS IN KTH ACTIVITY(BILBN)

   69 L=0
      P=0
      BILBN(L)=0.0
      DO 951 K=1,NOACT
      DO 952 J=1,45
      P=P+1
      DO 953 I=1,45
      IF(AMATX(I,P).NE.1) GO TO 953
      L=L+1
      BILBN(L)=BILLS(J,K)*PRI(K)
      ROW(L)=P
      GO TO 952
  953 CONTINUE
  952 CONTINUE
  951 CONTINUE
      WRITE(6,95)((ROW(M),BILBN(M)),M=1,L)
   10 FORMAT (8X,1F5.3)
   20 FORMAT (10X,45I1,9X,I5)
   37 FORMAT(5X,10F6.0)
   30 FORMAT (5X,10I5)
   36 FORMAT(5X,10F5.3)
   11 FORMAT( 'NAME',T15,'PERS')
   21 FORMAT('ROWS')
   31 FORMAT(T2,'N',T5,'OBJ')
   43 FORMAT(T2,'L',T5,'ACT',T8,I5)
   51 FORMAT(T2,'E',T5,'OFFTYP',T11,I2)
   55 FORMAT(T2,'E',T5,'ROW',T8,I5)
   71 FORMAT('COLUMNS')
   81 FORMAT(T5,I8,T15,'OBJ',T25,F8.3,T40,'ACT',T43,I5,T50,'1'/
     1T5,I8,T15,'OFFTYP',T21,I2,T25,'1',T40,'ROW',T43,I5,T50,'1')
   91 FORMAT(T5,I6,T15,'OBJ',T25,F8.3,T40,'ROW',T43,I5,T50,'1')
   96 FORMAT(T5,'Y',T6,I6,T15,'ROW',T18,I5,T25,'-1')
   97 FORMAT(T5, 'Y',T6,I4,T15,'OBJ',T25,F8.3,T40,'OFFTYP',T46,I2,
     1T50,'1')
   92 FORMAT('RHS')
   93 FORMAT(T5,'PERS',T15,'ACT',T18,I5,T25,F8.2,T40,'ACT',T43,I5,
     1T50,F8.2)
   98 FORMAT(T5,'PERS',T15,'ACT',T18,I5,T25,F8.2)
   94 FORMAT(T5,'PERS',T15,'OFFTYP',T21,I2,T25,F8.2,T40,'OFFTYP',
     1T46,I2,T50,F8.2)
```

```
88 FORMAT(T5,'PERS',T15,'OFFTYP',T21,I2,T25,F8.2)
95 FORMAT(T5,'PERS',T15,'ROW',T18,I5,T25,F8.2,T40,'ROW',T43,I5,
  1T50,F8.2)
   STOP
   END
```

LIST OF REFERENCES

1. Johnson, Ronald L. and Ronald D. Newmister, The Naval Officer Assignment Problem, Masters' Thesis Naval Postgraduate School, Monterey, California, June 1967.

2. Daniels, James M., A Planning Model for the Optimum Utilization of Manpower Resources, Masters' Thesis, Naval Postgraduate School, Monterey, California, June 1967.

3. Hillier, F. S. and G. J. Lieberman, Introduction to Operations Research, p.198-204, Holden-Day, Inc., 1967.

4. Hadley, G., Linear Programming, p.367-368, Addison-Wesley Publishing Co., Inc., 1962.

5. Personnel Laboratory, Wright Air Development Center, Air Research and Development Command, United States Air Force, Lackland Air Force Base, Texas Technical Report WADC-TR-59-359, The Mathematics of Personnel Utilization Models, by C. F. Kossack and R. E. Beckwith, November 1959.

6. Management Sciences Research Group, Carnegie - Mellon University, Pittsburg, Pennsylvania. Report No. NONR 760(24), NR 047-048, Multi-dimensional and Dynamic Assignment Models with Some Remarks on Organization Design, by A. Charnes, W. W. Cooper and A. Stedry, March 1968.

7. Fishburn, P. C., Decision and Value Theory, p.77-130, Wiley, 1964.

8. Gomory, R. E., An Algorithm for Integer Solutions to Linear Programming, Princeton-IBM Mathematics Research Project, Technical Report No. 1, November 1958.

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13. ABSTRACT

The Officer Distribution Plan (ODP) has been developed by the Navy in an attempt to provide for the most equitable distribution of officer resources throughout the naval establishment. This thesis demonstrates the feasibility of constructing a linear programming model to represent the U. S. Navy officer personnel system for the purpose of developing the ODP and which can do in minutes what now takes months. Constraints in the model represent officer category limitations, activity configuration and billet eligility. A measure of billet fill effectiveness is developed through the assignment of penalty cost for perfect fills, imperfect fills, unassigned resources and unfilled and overfilled billets based on inventory, requirements and activity priority rating. Two example problems using fiscal year 1969 data are solved to illustrate the technique. The author recommends that a full scale test problem be constructed with an activity configuration which resembles that of the current ODP in order to better estimate the value of this approach to officer personnel planning.

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Owens

A linear programming model for naval officer distribution.